

BRK-011 · From electrodynamic infinities to Wilson's scales

BRK · Conceptual Breaks

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An infinity in a calculation is not an experimental result. It is a warning that the relation between a mathematical description and a measured quantity has not yet been brought under control. In twentieth-century field theory, renormalization changed from a way of obtaining finite predictions in a difficult theory into a broader way of investigating how descriptions change with scale.

That development did not follow a straight line from an embarrassing trick to an inevitable final interpretation. Problems in quantum electrodynamics and problems near thermodynamic critical points arose in different experimental settings. Their eventual connection changed the questions that physicists could ask of a theory, as well as the techniques available for answering them.

1. A discrepancy small enough to measure, large enough to matter

The late-1940s problem was not simply that calculations contained infinities. Atomic measurements also demanded small, definite corrections to simpler predictions. Bethe's 1947 paper calculated an electromagnetic contribution to the shift of hydrogen energy levels. It used an approximate treatment, rather than a complete relativistic solution. [1]

The logic of such a calculation depends on distinguishing an observable energy difference from a parameter before it has been related to measurement. A formula may contain both a correction

and a quantity called the electron mass. Unless the latter has a specified relation to the experimentally measured mass, adding the correction can count part of the same physical effect twice.

This is why subtracting a divergent contribution is not automatically either legitimate or illegitimate. The question is what has been defined, what has been measured and which relations remain predictive after those definitions. An arbitrary subtraction fitted separately to every observation would explain little. A consistent prescription fixing a small set of quantities can constrain many other results.

2. Precision made a calculational program testable

Schwinger's 1948 calculation addressed a different observable, the electron's magnetic moment. Its leading correction to the simplest Dirac result is expressed by the anomalous-moment coefficient $\alpha/(2\pi)$, where α is the fine-structure constant. This connected the theoretical treatment to a finite measurable departure, rather than to an infinity considered in isolation. [2]

Using the illustrative approximation $\alpha = 1/137$ gives $\alpha/(2\pi) \approx 0.001162$. This is the leading contribution to the dimensionless anomaly $(g - 2)/2$, not the full modern prediction and not the value of g itself. Confusing this anomaly with $g - 2$ would give a factor-of-two error. Confusing it with g itself, approximately 2.002323 at this order, would be a different and much larger error.

Higher-order terms remain to be calculated. Agreement at one order does not prove convergence of the perturbative series or completeness at arbitrarily short distances.

3. More than one route to the same predictions

In his 1965 Nobel lecture, Tomonaga describes the work of his group alongside that of Schwinger and Feynman, and Dyson's demonstration that the renormalization procedure handles the divergences to arbitrarily high perturbative order. This is a participant's retrospective account, valuable for its description of the technical program but not a neutral census of every contemporary response. [3]

The achievement of 1947–1949 was a systematic treatment extending beyond one specially chosen correction. Relativistic consistency and calculational bookkeeping mattered because the same theory had to support multiple predictions.

It is misleading to describe this solely as physicists deciding to ignore infinity. Regularization introduces a controlled intermediate prescription; renormalization relates parameters and predictions to specified physical conditions. Whether the regulator can be removed, and in what sense, is a further question. Keeping these operations distinct explains why success could coexist with concern about foundations.

4. The missing intermediate question: which charge, at which scale?

Finite predictions did not end the question of how a renormalized theory was defined. In 1953 Stueckelberg and Petermann treated transformations between different finite reparametrizations of QED. Wilson identifies this work as an early formulation of the renormalization group. A theory could be represented by different parameter choices without making those choices different physical theories. [4]

Gell-Mann and Low's 1954 paper investigated QED at small distances. They related the large-momentum behavior of propagation functions to renormalization constants and examined the charge distribution around a test charge. The problem was not another isolated atomic correction: it concerned how the same theory behaves as the distance being probed changes. [5]

Wilson explains the bridge in terms of a family of charge parameters defined at different momentum scales, with a differential equation relating them. He identifies that equation as a predecessor of his own. The link therefore concerns the transformation between descriptions of one theory, not a resemblance between two uses of the word "renormalization." [4]

This made an ambiguity in parametrization productive. Instead of asking only which finite value should replace a bare parameter, one could ask how values defined at different reference scales are connected. Physical predictions must be related consistently across those choices. A measured charge at long distance cannot simply be treated as an unexamined input at every shorter distance.

5. A field-theory problem meets critical phenomena

Wilson's route began in particle field theory. In his retrospective account, a model divided into well-separated momentum slices let him eliminate a high-energy slice and obtain a modified Hamiltonian for the remaining slices. That construction supplied a concrete basis for successive renormalization. His encounter with Kadanoff's 1966 scaling picture then connected this work to critical phenomena. [4]

The 1971 paper explicitly developed the Kadanoff picture using renormalization-group methods. Critical phenomena supplied a setting where fluctuations across many scales had to be treated together, rather than as a small correction to an average descrip-

tion. The transformation of the model became a calculational object. [6]

These stages should not be collapsed into one discovery. Changing finite QED parametrizations, studying short-distance behavior and eliminating degrees of freedom from a model are related operations with different starting problems. Their historical connection is stronger when those differences remain visible. It is weaker when the whole development is retold as a generic lesson that large and small scales look different.

The conceptual account of fixed points and universality belongs in IDE-006. Here the important change is the movement of a technical question between research settings: how does altering the resolution change the parameters and interactions that must be retained? The earlier QED problem supplied part of the mathematical lineage; critical phenomena made the scale transformation a systematic route to new predictions.

6. What changed in the burden of explanation

Relations between bare and measured parameters, between renormalization scales and between successive effective models are distinct. Their historical connection was neither a single subtraction trick nor a sudden invention in 1971.

The finite atomic calculations remain important evidence for the earlier program. The later scale analysis added a way to investigate why a description works over a range and how its parameters must change when that range changes. It did not establish convergence of every perturbative expansion or turn every divergent theory into a consistent one.

The break was thus a change in what could be studied: the dependence of a theoretical description on its scale could itself be made

into an organized problem. Wilson's participant account documents one route into that problem. It should not be mistaken for evidence that all contemporaries understood the development in the same way or accepted it at once.

Related reading

BRK Article 010, "Deterministic laws cease to guarantee practical prediction," distinguishes the existence of governing equations from the conditions needed for useful prediction.

References

- [1] Bethe, H. A. "The Electromagnetic Shift of Energy Levels." *Physical Review* 72, 339–341 (1947). <https://doi.org/10.1103/PhysRev.72.339>
- [2] Schwinger, J. "On Quantum-Electrodynamics and the Magnetic Moment of the Electron." *Physical Review* 73, 416–417 (1948). <https://doi.org/10.1103/PhysRev.73.416>
- [3] Tomonaga, S. "Development of Quantum Electrodynamics." Nobel Lecture (1965). <https://www.nobelprize.org/prizes/physics/1965/tomonaga/lecture/>
- [4] Wilson, K. G. "The Renormalization Group and Critical Phenomena." Nobel Lecture (1982). <https://www.nobelprize.org/prizes/physics/1982/wilson/lecture/>
- [5] Gell-Mann, M., and Low, F. E. "Quantum Electrodynamics at Small Distances." *Physical Review* 95, 1300–1312 (1954). <https://doi.org/10.1103/PhysRev.95.1300>
- [6] Wilson, K. G. "Renormalization Group and Critical Phenomena. I. Renormalization Group and the Kadanoff Scaling Picture." *Physical Review B* 4, 3174–3183 (1971). <https://doi.org/10.1103/PhysRevB.4.3174>