

BRK-010 · Deterministic laws cease to guarantee practical prediction

BRK · Conceptual Breaks

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A deterministic equation specifies how a state evolves. It does not promise that the state can be measured with unlimited accuracy, or that a small measurement error will remain small. Separating these claims changed what it meant to understand a physical system. A law could remain definite while a detailed forecast became unreliable.

This was not a single discovery that overturned a universally accepted belief in perfect prediction. Earlier mathematical work had already exposed difficulties in the behavior of trajectories. The later development of chaos research made those difficulties concrete in relatively simple models, and made sensitivity itself an object of calculation.

1. The missing condition in a promise of prediction

Poincaré's discussion of chance in *Science et méthode* distinguished exact dependence on initial conditions from practical predictability. He described how a small initial discrepancy could become a large later discrepancy. The significance is the conditional character of prediction: approximate knowledge of the present gives approximate knowledge of the future only when the dynamics preserves the relevant accuracy. [1]

A familiar stable system helps clarify the contrast. Two slightly different starting displacements of a damped oscillator can

converge toward the same resting state. Initial uncertainty is then progressively less important for a question about its final position. Determinism alone did not establish that convergence; the particular dynamics did.

Now imagine an evolution that repeatedly stretches a small region of possible states. Even if each exact state has a unique future, the collection of futures allowed by the initial uncertainty can spread. The investigator has not lost the governing rule. The rule has transformed a modest observational uncertainty into a serious predictive limitation.

2. A small atmospheric model with an unexpected burden

Lorenz's 1963 paper studied a set of deterministic differential equations motivated by convection. Its numerical solutions showed bounded, nonperiodic behavior that was sensitive to small changes in initial state. It was a deliberately reduced mathematical system, not a full operational model of the atmosphere. [2]

That distinction matters in both directions. Because the system was small, its irregular behavior could not simply be blamed on an enormous inventory of unknown external causes. But because it was reduced, its precise numerical behavior could not be assigned directly to real weather. It established a possibility and a mechanism in a specified model.

The computer was not merely a faster calculator of a familiar answer. Repeated integration let trajectories be inspected over intervals in which divergence became apparent. Yet numerical calculation also introduced a question that any convincing interpretation had to face: was the irregularity a property of the equations, or an artifact of approximation and rounding?

An apparently wandering graph is not enough to answer that question. One must examine stability, numerical methods and the geometry of the evolution. The conceptual advance was to investigate irregularity within the deterministic system rather than assume in advance that it must represent a missing random input.

3. What an error-growth estimate says

For a simple local illustration, suppose a small uncertainty grows approximately as $\delta(t) = \delta(0) \exp(\lambda t)$, with λ positive. This expression is an approximation over a regime where exponential separation is a useful description. It is not a universal law of every perturbation at every time.

If the maximum useful error is Δ , the resulting forecast horizon is approximately $\ln[\Delta/\delta(0)]/\lambda$. Improving the initial measurement helps, but its benefit is logarithmic in this model. Reducing the initial error by a factor of ten adds $\ln(10)/\lambda$ to the horizon; it does not multiply that horizon by ten.

For illustrative values $\lambda = 0.5$ per day and a tolerance-to-initial-error ratio of 100, the horizon is about 9.21 days. Improving the initial precision tenfold makes the ratio 1,000 and the horizon about 13.82 days. These are hypothetical inputs, not estimated constants of terrestrial weather.

The calculation separates three contributions to a forecast: the uncertainty at initialization, the rate at which it grows and the accuracy the intended use requires. There is no single horizon independent of the question. A forecast of a broad probability distribution can remain useful after a precise trajectory has lost usefulness.

4. Simple rules need not have simple long-term behavior

May's 1976 discussion of nonlinear difference equations showed why a short recurrence relation need not produce a short catalog of behaviors. Depending on its parameters, a simple map can approach a fixed point, cycle among several values or display chaotic behavior. [3]

Consider the illustrative logistic map $x_{\text{next}} = r x(1 - x)$, with x between zero and one and a suitable positive r . The formula is deterministic: give it an exact current value and parameter, and it gives one next value. That fact does not tell us whether nearby starting values stay near, or whether long iteration converges to a fixed point.

The map is not evidence that every population follows a logistic recurrence, still less that every social development is chaotic. Applying it to a real system requires justification for the state variable, the update interval and the neglected processes. A useful demonstration of mathematical possibility can become misleading when converted into an unsupported universal explanation.

5. Irregularity, randomness and evidence

Deterministic chaos is not the same claim as quantum randomness. One concerns the behavior of trajectories generated by specified dynamics; the other concerns the probabilistic structure and interpretation of quantum outcomes. Calling both unpredictable does not make their mechanisms interchangeable.

Nor can every irregular experimental time series be classified as chaotic by appearance. Measurement noise, changing external forcing and an inadequate model can generate complicated records. Evidence for deterministic chaos requires more than

resemblance to a celebrated plot. The proposed dynamics must account for features that rival explanations do not equally explain.

Chaos also leaves some questions tractable. A system may have a useful statistical description even when individual trajectories diverge rapidly. The failure of one kind of prediction can motivate a different kind, provided the statistical regularities are established rather than assumed.

6. The new explanatory obligation

After this conceptual change, presenting a deterministic equation was no longer enough to justify an indefinitely detailed forecast. The investigator also owed an account of initial uncertainty, stability, approximation and the observable being predicted. The law and the forecast had become distinct achievements.

This did not reduce explanation to helplessness. It made explanation more specific. A successful theory could describe why detailed forecasts fail, how their useful horizon changes and which coarser properties remain predictable. The important break was the loss of an unjustified guarantee, accompanied by new ways to calculate the limits of knowledge.

Related reading

BRK Article 011, “From electrodynamic infinities to Wilson’s scales,” follows another case in which the conditions under which a calculation is meaningful became part of the scientific explanation.

References

- [1] Poincaré, H. *Science et méthode* (1908), Book I, chapter IV, “Le hasard.” https://fr.wikisource.org/wiki/Science_et_m%C3%A9thode

- [2] Lorenz, E. N. "Deterministic Nonperiodic Flow." *Journal of the Atmospheric Sciences* 20, 130–141 (1963). [https://doi.org/10.1175/1520-0469\(1963\)020%3C0130:DNF%3E2.0.CO;2](https://doi.org/10.1175/1520-0469(1963)020%3C0130:DNF%3E2.0.CO;2)
- [3] May, R. M. "Simple mathematical models with very complicated dynamics." *Nature* 261, 459–467 (1976). <https://doi.org/10.1038/261459a0>