

Calculus, limits and infinitesimals

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Core question

How did seventeenth-century work on tangents, areas, motion and infinite series become the differential and integral calculus, and why were its foundations repaired only later?

Historical and evolutionary context

Problems of area, tangency, motion and accumulation had been attacked for centuries. Greek exhaustion methods could establish rigorous results for particular areas and volumes, and medieval and early-modern mathematicians developed sophisticated work on indivisibles, series and kinematics. What was missing was a general symbolic machinery that could treat change and accumulation as systematically related operations.

What could communities know before this transition?

Before Newton and Leibniz, Greek exhaustion methods, medieval kinematics, Cavalieri's indivisibles, Fermat's work on tangents and extrema, Barrow's geometric results and the Kerala school's infinite-series techniques had already solved important special problems. What was missing in Europe was a compact general symbolism and set of operations linking rates of change and accumulation across many problems.

What changed?

Two seventeenth-century constructions

Newton developed methods of fluxions during the 1660s, thinking of quantities as generated by continuous motion and of their rates as fluxions. Much of this work remained unpublished for years. Leibniz independently developed a differential and integral notation and published a short calculus paper in 1684, followed by work on integration in 1686. His notation, including the differential d and integral sign, proved especially adaptable for communication and later analysis.

The priority dispute between Newton and Leibniz became nationalised and bitter. Modern historical treatment recognises independent development rather than reducing the subject to a single inventor. The two traditions also differed in notation, style and networks of followers.

From techniques to a general language

Calculus linked two classes of problems: instantaneous change and accumulated quantity. The fundamental relation between differentiation and integration made many geometric and physical problems transformable into one another. This was an epistemic change because it converted local tricks into a reusable representational system.

But the early foundations were not fully secure. Infinitesimals and infinitely small increments worked powerfully in calculation while remaining philosophically and mathematically contested. Eighteenth-century analysts extended the techniques far faster than they clarified their foundations.

Limits and the later repair of foundations

The modern epsilon-delta concept of limit belongs mainly to nineteenth-century analysis, not to Newton or Leibniz. Cauchy, Weierstrass and others later supplied more explicit definitions of continuity, convergence and limit. PIO therefore treats “limits” as the later foundational consolidation of a seventeenth-century computational revolution.

This matters because scientific tools often become productive before their conceptual foundations are fully stabilised. Calculus could solve mechanics and astronomy problems while debates over infinitesimals continued.

Relation to Newtonian mechanics

Newtonian mechanics and calculus developed in parallel and interacted deeply, but they should not be collapsed. The *Principia* of 1687 was written largely in geometric form. Later mechanics, celestial perturbation theory, fluid dynamics and mathematical physics

increasingly used differential equations and analytic methods that transformed Newton's framework.

Why could this capability emerge?

A dense seventeenth-century problem culture around astronomy, mechanics, optics, curves and infinite series made a general calculus valuable. Fermat, Barrow and Cavalieri were direct European precursors, while the Kerala school associated with Madhava had developed sophisticated infinite-series results from the fourteenth century onward; no secure historical transmission from Kerala to Newton or Leibniz has been demonstrated.

What later forms of knowledge did it enable?

Calculus supplied a general language for mechanics, astronomy, fluids, fields, probability and later differential equations. Nineteenth-century work on limits and rigor then made explicit assumptions that seventeenth-century users had often handled through geometric intuition or infinitesimals.

New limits and risks

The new calculus could solve problems of motion, tangency, quadrature and accumulation faster and more generally than older geometric methods, but its early foundations were not secure. Infinitesimals, fluxions and limiting procedures were often manipulated successfully before mathematicians had a shared account of why those operations were valid, creating room for hidden assumptions and formally persuasive but unreliable arguments. Nineteenth-century work on limits and rigor reduced these ambiguities without erasing the historical fact that useful technique preceded foundational clarity.

What changed in the architecture of knowledge?

Mathematics gained a **general operator language for continuous change**. Motion, growth, curvature, area and accumulation could be represented within a common formal system. This made whole families of physical problems calculable and created a bridge from geometry to differential equations.

Loss, exclusion and uncertainty

Standard histories long treated calculus as an exclusively Newton-Leibniz origin story. That framing can obscure Fermat, Barrow, Cavalieri and especially the earlier Kerala series tradition associated with Madhava. The historical correction is not to invent an undocumented transmission route, but to recognise multiple lines of sophisticated work on infinity, series, quadrature and change.

Dependency map

- RETROSPECTIVE ← PIO-024 Eudoxus, Archimedes and exhaustion methods
- RETROSPECTIVE ← PIO-033 Al-Khwarizmi and algebraic procedures
- INDIRECT ← PIO-040 How numerals travelled: India, Baghdad and Pisa
- PARALLEL ← PIO-053 Newton, mechanics and universal gravitation
- INDIRECT → PIO-063 Thermodynamics and energy accounting
- INDIRECT → PIO-067 Maxwell and electromagnetic unification
- INDIRECT → PIO-076 Einstein and spacetime

References

1. Leibniz, G. W. "Nova Methodus pro Maximis et Minimis..." *Acta Eruditorum* (1684).
2. Newton, I. *De Analysi per Aequationes Numero Terminorum Infinitas* (written 1669; published later).
3. Guicciardini, N. *The Development of Newtonian Calculus in Britain, 1700-1800*. Cambridge University Press (1989).
4. Boyer, C. B. *The History of the Calculus and Its Conceptual Development*. Dover reprint (1959; original 1949).