

KNO-017 · Is Mathematics a Language?

KNO · Ways of Knowing

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Ways of Knowing

Epistemic profile: Current scientific interpretation **Licence at publication:** All Rights Reserved

Article Abstract

Mathematics is often called a language, but what exactly is being compared when we say that? This article asks whether mathematical notation, formal systems, proof practices, abstraction and applied modelling satisfy the same functions as natural language, or whether the analogy works only for selected parts of mathematics. It examines syntax, semantics, interpretation, proof, calculation and external cognitive representation while separating mathematical expressions from the wider practices in which they are created and justified. It also asks why formal symbols can support unusually precise manipulation without carrying meaning by themselves, and why the effectiveness of mathematics in science does not settle what mathematics ultimately is. The question is therefore not whether mathematics uses languages, but how far the language analogy can be pushed before it hides important differences.

In 60 Seconds

Mathematics contains formal languages, but mathematical practice is broader than symbol strings. Formal notation has syntax: rules determine which expressions are well formed and which transformations are permitted. Meaning enters through interpretation, definitions, conventions and connections to mathematical structures or empirical systems. Proof is not merely the presence of symbols; it is a rule-governed argument whose validity depends on the system in which it is read. Notation can still transform cognition by making relations easier to inspect and manipulate. Abstraction lets the same structure apply across very different domains, while calculation can produce correct outputs without supplying an explanation of why a model is appropriate. In science, mathematics can represent relations with unusual precision, yet its success does not by itself decide whether mathematics is discovered, invented or literally a language. The useful conclusion is conditional: language is a powerful analogy for some mathematical components, but not a complete account of mathematical knowledge.

Why This Question Matters

Calling mathematics a language can illuminate how notation, syntax and shared conventions make reasoning portable, but the metaphor can also conceal important differences between formal manipulation, proof, meaning and empirical interpretation. The issue matters because mathematics is central to nearly every quantitative science, and its symbols often look more self-sufficient than they are. Understanding what notation contributes—and what it cannot supply by itself—helps separate mathematical validity from modelling adequacy, calculation from explanation and formal precision from claims about the world.

1. There Is More Than One Sense of “Language”

Natural languages combine syntax, semantics, pragmatics and social convention. Formal languages deliberately isolate some of these features. In classical logic, a formal language can be specified as a recursively defined set of strings over an alphabet, while deductive rules and semantics provide inferential or interpretive structure [S01].

Mathematics uses many such formal or semi-formal systems, but it also uses natural language and diagrams. The phrase “mathematics is a language” is therefore not a single technical thesis. It can refer to notation, to formal syntax, to proof, to communication, or to representation.

[C01] Parts of mathematics can be represented as formal languages with explicit syntax and inferential rules, but this does not by itself establish that all mathematical practice is one formal language. [S01,S02]

Where That Leaves Us

Natural languages combine syntax, semantics, pragmatics and social convention. Mathematics uses many such formal or semi-formal systems, but it also uses natural language and diagrams. In classical logic, a formal language can be specified as a recursively defined set of strings over an alphabet, while deductive rules and semantics provide inferential or interpretive structure [S01]. The next section examines the proposition that syntax is not yet meaning.

2. Syntax Is Not Yet Meaning

A string can be well formed without yet specifying what its symbols denote. Logic textbooks distinguish syntax from semantics precisely because a formal expression and an interpretation of that expression are different objects [S01].

This matters scientifically. The equation $y = ax + b$ can describe many systems, or none, depending on how its symbols are mapped to quantities and conditions. Formal validity therefore does not certify empirical interpretation.

[C02] Well-formed mathematical syntax does not by itself determine the empirical meaning of the symbols being manipulated. [S01]

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A string can be well formed without yet specifying what its symbols denote. Formal validity therefore does not certify empirical interpretation. The equation $y = ax + b$ can describe many systems, or none, depending on how its symbols are mapped to quantities and conditions. The next section examines the proposition that notation changes what can be seen and manipulated.

3. Notation Changes What Can Be Seen and Manipulated

Notation is not a transparent wrapper around pre-existing thought. A good notation makes some structures easier to compare, transform and reuse. Historical work on mathematical notation shows how symbolic compression can expose patterns that are cumbersome in prose [S03].

This does not mean notation creates mathematical truth. It means representations alter the cognitive and practical accessibility of relationships. Algebraic notation, coordinate systems and operator notation make different operations cheap or expensive to perform.

[C03] Mathematical notation can reorganise the accessibility of relationships and operations even when the underlying mathematical content is unchanged. [S03]

Where That Leaves Us

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4. Formalisation Makes Rules Inspectable

Formalisation replaces many tacit conventions with explicit grammars, definitions and inference rules. Mathematical logic made it possible to study mathematical theories themselves as formal objects [S02,S04]. Modern proof assistants extend that idea computationally: definitions and proofs can be encoded in a formal language that a machine parses and checks against a foundational system [S05].

The gain is auditability, not omniscience. Formal verification checks a derivation relative to the encoded definitions, axioms, libraries and software environment. It does not decide whether a scientific formalisation chose the right phenomenon to represent.

[C04] Formalisation can make syntactic and inferential commitments mechanically inspectable, while leaving the choice and interpretation of the formalisation as a separate problem. [S04,S05]

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5. Mathematical Proof Is More Than Printed Symbol Strings

Mathematical practice usually communicates proofs in a compressed social form. Published arguments omit elementary transformations, rely on shared definitions and often mix prose, equations and diagrams. Contemporary philosophy of mathematical practice therefore distinguishes formal derivations from the informal or semi-formal proofs mathematicians actually write and evaluate [S06].

A proof can consequently have two related standards: whether its conclusion follows under the accepted mathematical rules, and whether the presentation makes the relevant reasoning intelligible and checkable to its audience.

[C05] Ordinary mathematical proofs are commonly presented in natural language interspersed with formal symbols rather than as fully expanded formal derivations. [S06]

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distinguishes formal derivations from the informal or semi-formal proofs mathematicians actually write and evaluate [S06]. The next section examines the proposition that formal languages can have many interpretations.

6. Formal Languages Can Have Many Interpretations

One reason formal systems are reusable is that the same syntax can admit different models or interpretations. This is a strength, but it blocks a simple identification between an equation and one physical reality.

In applied science, a mathematical structure becomes a model of a target system only after variables, parameters, boundary conditions and correspondence relations are specified [S01,S07]. KNO-013 treats model adequacy; here the narrower point is representational: mathematical form underdetermines application.

[C06] Reuse of the same mathematical structure across domains depends on interpretation; the formal structure alone does not select its scientific target. [S01,S07]

Where That Leaves Us

One reason formal systems are reusable is that the same syntax can admit different models or interpretations. This is a strength, but it blocks a simple identification between an equation and one physical reality. KNO-013 treats model adequacy; here the narrower point is representational: mathematical form underdetermines application. The next section examines the proposition that abstraction is a feature, not a defect.

7. Abstraction Is a Feature, Not a Defect

Mathematics often suppresses material detail in order to preserve structure. The same group, graph, differential equation or probability space can be instantiated by systems with very different physical composition. This abstraction supports transfer of methods between domains.

But abstraction also creates a gate: the omitted features must genuinely be irrelevant to the question being asked. The mathematical derivation may be exact inside the abstraction while the abstraction is scientifically inadequate.

[C07] Mathematical abstraction supports transfer by preserving selected structure across different domains, but scientific adequacy depends on whether the omitted features are irrelevant to the target question. [S02,S07]

Where That Leaves Us

Mathematics often suppresses material detail in order to preserve structure. But abstraction also creates a gate: the omitted features must genuinely be irrelevant to the question being asked. This abstraction supports transfer of methods between domains. The next section examines the proposition that mathematics can express relations more precisely than prose.

8. Mathematics Can Express Relations More Precisely Than Prose

Natural language can state that one quantity rises with another; an equation can specify the functional relation, parameter dependence and limiting cases. Units, domains and assumptions can be made explicit. This precision is one reason mathematics is indispensable in mature quantitative sciences.

Yet precision of expression must not be confused with precision of knowledge. A model can contain many significant digits while its parameters are poorly measured or its structure is wrong. KNO-015 supplies the uncertainty boundary.

[C08] Mathematical representation can increase precision about relations and assumptions without guaranteeing that the represented quantities or model are empirically accurate. [S07]

Where That Leaves Us

Natural language can state that one quantity rises with another; an equation can specify the functional relation, parameter dependence and limiting cases. Yet precision of expression must not be confused with precision of knowledge. This precision is one reason mathematics is indispensable in mature quantitative sciences. The next section examines the proposition that calculation and explanation are not identical.

9. Calculation and Explanation Are Not Identical

A derivation can produce a correct numerical result without explaining why the phenomenon occurs. Conversely, an explanatory argument may depend on qualitative structure rather than a long calculation. Philosophers of mathematics and science therefore distinguish proof, calculation, prediction and explanation rather than treating them as synonyms [S06,S07].

The distinction matters for communication. “The equation gives the right number” is evidence of empirical adequacy in a tested domain, not automatically an explanation of mechanism.

[C09] Correct calculation, valid proof, successful prediction and scientific explanation are related but non-identical epistemic achievements. [S06,S07]

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A derivation can produce a correct numerical result without explaining why the phenomenon occurs. Conversely, an explanatory argument may depend on qualitative structure rather than a long calculation. Philosophers of mathematics and science therefore distinguish proof, calculation, prediction and explanation rather than treating them as synonyms [S06,S07]. The next section asks why mathematics works so well in science.

10. Why Does Mathematics Work So Well in Science?

Wigner famously framed the effectiveness of mathematics in the natural sciences as something that itself demands explanation [S08]. His essay is historically important because it identifies a genuine puzzle: mathematical structures developed with substantial autonomy often later find striking scientific applications.

But the observation does not settle what mathematics is. It does not prove Platonism, formalism, structuralism or the claim that the universe literally “is mathematics”. Those are philosophical positions that require additional arguments.

[C10] The successful application of mathematics to natural science is an empirical and philosophical fact to be explained, not by itself a proof of any single ontology of mathematics. [S02,S08]

Where That Leaves Us

Wigner famously framed the effectiveness of mathematics in the natural sciences as something that itself demands explanation [S08]. But the observation does not settle what mathematics is. It does not prove Platonism, formalism, structuralism or the claim that the universe literally “is mathematics”. The next section examines mathematics as external cognitive infrastructure.

11. Mathematics as External Cognitive Infrastructure

Written mathematics lets reasoning persist outside a single working memory. Expressions can be copied, transformed, checked and shared. A notation can preserve intermediate states so that a long inference does not have to be mentally reconstructed from scratch [S03,S05].

This connects KNO-017 to KNO-022, which will treat computation as external memory more generally. Here the narrower claim is that symbolic mathematics is an external representational technology as well as an abstract discipline.

[C11] Mathematical inscriptions and formal representations can function as external, revisable states in extended chains of reasoning. [S03,S05]

Where That Leaves Us

Written mathematics lets reasoning persist outside a single working memory. A notation can preserve intermediate states so that a long inference does not have to be mentally reconstructed from scratch [S03,S05]. Here the narrower claim is that symbolic mathematics is an external representational technology as well as an abstract discipline. The next section synthesizes the answer to the article's central question.

12. The Answer: Mathematics Contains Languages, but Is Not Exhausted by Them

The strongest defensible answer is therefore conditional. Mathematics contains formal languages and depends heavily on learned symbolic conventions. These languages provide compact representation and explicit inferential structure. But mathematics as practiced also includes interpretation, informal reasoning, diagrams, examples, proof strategies and judgments about relevance.

Calling mathematics “a language” is useful when it highlights representation and rule-governed inference. It is too strong when it collapses all mathematics into syntax or treats symbols as self-interpreting.

[C12] “Mathematics is a language” is defensible as a methodological metaphor for its representational and rule-governed systems, but not as an exhaustive description of mathematical practice. [S01,S02,S06]

Where That Leaves Us

The strongest defensible answer is therefore conditional. Mathematics contains formal languages and depends heavily on learned symbolic conventions. These languages provide compact representation and explicit inferential structure. But mathematics as practiced also includes interpretation, informal reasoning, diagrams, examples, proof strategies and judgments about relevance. Calling mathematics “a language” is useful when it highlights representation and rule-governed inference. The next section defines the boundary of KNO-017.

13. Boundary of KNO-017

KNO-017 does **not** decide whether mathematical objects exist independently of minds, whether mathematical truth is discovered or invented, or which philosophy of mathematics is correct. Those questions belong to the conceptual IDE series.

It also does not treat diagrams and visual encodings in depth; KNO-018 does that. Nor does it treat computation as a general external-memory technology; KNO-022 does that.

The methodological result is narrower: mathematical notation, formal languages and proof are tools that make relationships representable, transformable, communicable and inspectable, but their application to the world requires interpretation and evidence beyond syntax.

Where That Leaves Us

KNO-017 does not decide whether mathematical objects exist independently of minds, whether mathematical truth is discovered or invented, or which philosophy of mathematics is correct. It also does not treat diagrams and visual encodings in depth; KNO-018 does that. This defines the article's boundary and prepares the final synthesis without extending the claim beyond what the preceding sections establish.

What We Know, and How

Mathematics uses formal languages with syntax and interpretation, and notation can make reasoning more inspectable and computationally tractable. Proof, meaning, modelling and explanation nevertheless depend on practices and interpretations that are not exhausted by symbol strings.

What We Still Don't Know

The language analogy does not settle philosophical questions about what mathematics is, why mathematical structures apply so effectively in science or whether mathematical entities are discovered, invented or described in some other way.

What Could Change Our Minds

The scope of the analogy should be reconsidered if a proposed account explains formal notation while failing to explain proof practice, interpretation, abstraction or applied modelling, or if a competing framework better captures those functions without treating all mathematics as linguistic.

If You Remember Only One Thing

Mathematics contains languages and uses them powerfully, but mathematical knowledge is broader than the formal symbol systems in which it is written.

Related Articles

- KNO-003 · From Marks to Symbols
- KNO-013 · Models, Maps and Reality
- KNO-016 · How Evidence Is Built: Putting the Tools Together

- KNO-018 · Images, Diagrams and Visualisation

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