

KNO-002 · Counting Before Numbers

KNO · Ways of Knowing

Juan Segurola

v0.1 · EN

All Rights Reserved

1. Quantity Before an Exact Number

The Question

If a person can choose the fuller basket without saying how many objects either basket contains, what numerical knowledge is present, and what is still absent? Humans, infants and many non-human animals can compare approximate numerosities. Performance depends on ratio: distinguishing eight from sixteen is easier than eight from ten. This approximate number system represents magnitude with limited precision; it does not by itself generate an exact symbol for every positive integer.[2,3] A second system can track a small number of individual objects with high precision, usually within a narrow capacity limit. Following one, two or three items is not the same operation as estimating a large crowd, and neither system alone provides the successor rule that makes the counting numbers indefinitely extensible.[2] Experiments with Mundurucu participants showed competent approximate addition and comparison despite a limited exact-number lexicon. Studies with Pirahã participants likewise found that performance remained strong when items could be matched directly, but accuracy fell when exact quantities had to be stored, transformed or reproduced across a delay. These results show that exact number words are tools for stabilising some tasks, not prerequisites for every perception of quantity.[4-6] The experiments do not rank languages or peoples by cognitive sophistication. They isolate task demands. When a collection remains visible, direct matching can carry the solution; when it disappears, a conventional word, gesture or material arrangement may preserve an exact state. The contrast is between representational resources, not between minds with and without quantity. [5-7]

Where That Leaves Us

Approximate magnitude, small-set tracking and exact symbolic number are related but distinct, and neither core system supplies the successor rule that makes counting indefinitely extensible. Before asking when people had numbers, a stronger question is which quantitative task they could solve, at what precision, and with which internal or external resources. The contrast these experiments isolate is between representational resources, not between minds.

2. One-to-One Correspondence Is Not Yet a Numeral

The Question

Can two sets be shown equal without identifying their cardinal number? One-to-one correspondence pairs every member of one collection with one member of another. A herder can give one pebble to each animal leaving an enclosure and remove one when each returns. If no pebble remains and no animal is unmatched, the sets correspond even if no word equivalent to “thirty-seven” was used.[1,13] This procedure preserves equality and detects surplus or absence. It does not necessarily represent the total independently of the objects or explain that all arrangements with the same cardinality share one abstract number. The relation may remain

practical and local: one marker per animal, vessel or obligation.[13,14] A tally introduces persistence. If one mark is deliberately mapped to one event, the record can be revisited after the event has passed. Yet repeated marks are not automatically tallies. Archaeology must establish intentional production and a plausible unit mapping before it can treat regularity as quantitative notation.[1] Even a genuine tally need not be a numeral system. Its marks may be interchangeable accumulations rather than named signs with conventional values. The tally answers “how many units have occurred here?” by allowing recounting; a numeral such as 37 encodes a value within a structured notation that can travel between contexts.[18]

Where That Leaves Us

Matching, tallying and naming an exact cardinal solve different problems. One-to-one correspondence may secure equivalence without a number word; a tally may preserve units without a portable numeral; a numeral system makes values conventional and reusable. Repeated marks are not automatically tallies, and deliberate production and a plausible unit mapping must be established before regularity can be read as quantitative notation.

3. Fingers, Bodies and Grouping

Body-based counting systems turn a stable anatomy into an ordered route. Fingers are especially convenient because they are visible, movable and naturally grouped, but documented systems also traverse wrists, elbows, shoulders, the head and the opposite side of the body. The sequence can serve as an externalised order even when its “signs” are enacted rather than detached objects. [11,12] Finger counting is not uniform. Cultures differ in starting hand, direction, finger order and whether a gesture denotes a cardinal value, an ordinal position or an action in a calculation. Those differences can influence numerical processing, which makes the hand a learned representational tool rather than a universal code waiting to be read.[11] Grouping reduces the burden of long one-by-one sequences. Collections can be organised in twos, fives, tens, twenties or other bundles; a higher-order unit then stands for several lower-order units. Bases often reflect body practices or material devices, but no single base follows inevitably from human anatomy.[18,20] An ordered count list becomes an exact-number technology only when learners coordinate several principles. Each item must receive one term, the stable order must be preserved, irrelevant changes of arrangement must not alter the total, and the final word must be understood as the cardinal value of the whole set. Children can recite a count list before they understand this cardinal principle. [9,10] Once the successor relation is understood, adding one item connects a set to the next number in the sequence. That insight supports generalisation beyond memorised small values. The history of a society’s number practices cannot be inferred directly from child development, but developmental experiments clarify which conceptual relations a mature counting system must coordinate.[9,10]

Where That Leaves Us

The body can supply order, memory and grouping without dictating one universal notation, and no single base follows inevitably from human anatomy. Exact counting emerges from learned coordination among a stable sequence, individual items and cardinal totals, not from the mere availability of ten fingers. Developmental evidence clarifies which relations a mature system must coordinate; it does not reconstruct any society’s history.

4. Tokens, Accounts and Institutions

Material counters can preserve quantity while freeing the represented objects to move elsewhere. A token may be handled, grouped, transferred or sealed; its usefulness depends on a rule connecting the object to a category, obligation or commodity. Material stability makes checking possible, but the rule remains social.[13,16] In southwest Asia, clay tokens occur long before the earliest written tablets. Denise Schmandt-Besserat proposed a long development from tokens to sealed envelopes, impressed signs and writing. The reconstruction is influential because it connects material forms with changing administrative needs, but the degree of direct continuity and its explanatory reach remain debated.[16] Late fourth-millennium BCE proto-cuneiform tablets provide firmer evidence for institutional numerical recording. Many record quantities and commodities using several metrological and numerical sign systems. A sign's value could depend on what was being counted or measured, so the same graphic form did not necessarily function as one context-free abstract number.[15,17] These tablets were not simply arithmetic notebooks. They belonged to administrative practices involving trained personnel, conventions, archives and recurrent transactions. Institution and notation co-developed: a durable sign system made accounts comparable, while repeated accounting supplied the settings in which signs could become standardised.[15] The regional sequence should not be inflated into a universal law. Other societies used knots, sticks, body sequences, oral formulae, written numerals and mixed devices. Similar tasks can recruit different media, and similar media can serve different tasks.[14,18]

Where That Leaves Us

The Near Eastern evidence documents one especially rich transition from material accounting to written numerical administration. It demonstrates what tokens and tablets could do in that history; it does not prove that every number system or every writing tradition began the same way. Institution and notation developed together there, and the degree of direct continuity from tokens to script remains debated rather than settled.

5. From Quantities to Numerals and Place Value

A numeral is a conventional sign or sign sequence for a number. Its efficiency depends on the rules of the notation: which forms are basic, how repetition works, whether position changes value, whether zero is available, and how units of different order are combined.[18] Additive systems repeat or combine value signs; multiplicative-additive systems express products of units; positional systems assign value partly through place. These are analytical categories, not evolutionary ranks. A notation can be highly effective for its uses without converging on modern decimal place value.[18] Place value compresses representation because a small set of digits can express arbitrarily large numbers when positions stand for powers of a base. It also creates a problem: an empty position must be distinguished from no notation at all. Placeholder marks and numerical zero have related but not identical histories.[18,20] Written numerals make values portable across objects and occasions. “37” can describe animals, days or lengths because the sign sequence is abstracted from the counted category. But practical systems often retain units, measures and commodity signs, reminding us that abstraction is achieved by convention rather than by erasing context automatically.[15,18] No single notation is the inevitable endpoint of counting. Roman numerals, Mesopotamian sexagesimal systems, Chinese counting rods, Mesoamerican vigesimal notation and Indo-Arabic decimal place value organise values differently. Comparison reveals multiple solutions to representation, calculation and transmission.[18]

Where That Leaves Us

Numerals are engineered conventions. Their forms encode choices about grouping, order, position and absence, and the categories that describe them are analytical rather than evolutionary ranks. Place value is powerful, but it is one historical solution within a diverse field, not the definition of numerical thought itself, and its abstraction is achieved by convention rather than by erasing context.

6. Measurement Is Not the Same as Counting

Counting assigns a number to discrete units under a rule for what counts as one. Measurement compares a continuous magnitude with a chosen unit. Five vessels can be counted; five litres result from repeatedly relating volume to a standard.[15,18] The distinction is not absolute at the procedural level because measurement often uses counting: one counts unit iterations, subdivisions or instrument marks. Conceptually, however, the operations differ. A counted total depends on individuating objects; a measured value depends on a unit, calibration and permissible precision.[15] Units are social and technical achievements. A cubit tied to a body reference becomes more reproducible when material standards and institutional procedures stabilise it. Standardisation allows results to travel, while also embedding authority in who maintains the reference and resolves discrepancies.[15] Measurement values can be fractional or approximate even when the counted instrument divisions are exact. Reporting a length therefore requires more than a numeral: it requires a quantity kind, a unit, a method and an uncertainty appropriate to the comparison.[18] Confusing counting with measurement can distort archaeology. A set of grouped signs may record discrete objects, grain capacity, labour time or an accounting conversion. The marks alone do not decide which quantity was represented.[15,17]

Where That Leaves Us

Counting concerns discrete units; measurement concerns comparison with a standard, and units are social and technical achievements rather than given facts. Historical practices intertwine them, but interpreting a record requires identifying the unit system and the represented magnitude, not merely reading a visible number. Grouped signs may stand for objects, capacity, labour time or an accounting conversion, and the notation by itself settles none of those readings.

7. What Archaeological Marks Can and Cannot Establish

Archaeological analysis can often establish material, manufacture, stroke order, spacing and whether groups were added in separate sessions. Those observations may support deliberate accumulation. They do not recover the counted category, the spoken labels or the rule connecting a mark to a unit.[1] Regularity is compatible with several purposes: decoration, grip, practice, ownership, rhythm, game, schedule or tally. Numerical interpretation becomes stronger when independent evidence links repeated marks to a recurrent countable process, when group boundaries fit a demonstrated rule, or when related objects preserve the same convention.[1] Modern observers can always count ancient marks. That fact belongs to the observer's notation, not automatically to the maker's practice. Claims about primes, calendars or arithmetic patterns are especially vulnerable to selection after the fact because many groupings can be discovered in a finite set of incisions.[1,14] Ethnographic analogy can generate hypotheses about how bodies, objects or marks support quantity. It cannot assign an ancient object to a living community's practice without historical connection. Similar solutions may arise independently, while superficially similar objects may participate in different conventions.[12-14] Preservation also filters the record. Perishable cords, wooden sticks, fibres, gestures and spoken sequences are less

likely to survive than bone or stone. The oldest surviving tally-like object is therefore not necessarily the first quantitative aid, and the absence of a durable notation does not demonstrate the absence of exact practice.[13,14]

Where That Leaves Us

The secure archaeological sequence is trace, manufacture, recurrence and context. Quantity, unit and meaning enter only when additional evidence supports the mapping, and the ability of a modern observer to count ancient marks belongs to that observer's notation rather than to the maker's practice. Preservation filters what survives, so the oldest durable tally-like object need not be the earliest quantitative aid.

8. Multiple Routes, Not One Staircase

Counting is best understood as a family of coordinated practices. Approximation compares magnitudes; small-set tracking follows individuals; correspondence matches collections; tallies preserve events; words stabilise ordered values; grouping creates higher units; numerals make values portable; measurement relates magnitudes to standards.[2,9,13,18] These capacities can reinforce one another without appearing in a fixed order. A community may possess exact oral counting with little durable notation, sophisticated measurement with specialised numerals, or material accounting tied closely to commodities. Historical change depends on tasks, teaching, media and institutions.[14,15,18] External devices do not merely store an already complete abstract number concept. By making groupings and correspondences visible, they can change which comparisons and operations are easy to learn and perform. At the same time, no object generates a numerical rule without users who maintain the mapping.[13,19] The diversity of number systems is therefore evidence, not noise. Different bases, body sequences, classifiers, sign values and calculation media reveal that numerical abstraction is constructed through repeatable relations rather than copied from one universal notation.[11,12,18] The most defensible origin story has no single starting object. Biological capacities for quantity are old; exact symbolic systems are cultural; surviving material records are selective; and locally documented transitions differ. The history of number is the history of how communities made quantitative relations stable enough to check, share and extend.[1,2,14]

Where That Leaves Us

There was no necessary march from notch to numeral to modern arithmetic. There were recurrent problems of matching, memory, grouping, exchange and measurement, solved through different combinations of bodies, language, materials and institutions. The diversity of number systems is evidence rather than noise, and the most defensible origin story has no single starting object.

What We Know, and How

Controlled experiments distinguish approximate magnitude, small-set tracking and learned exact-number practices. Developmental studies show that reciting count words precedes full understanding of cardinality. Cross-cultural studies demonstrate exact nonverbal performance under some conditions and the importance of words or external devices when quantities must be retained and transformed. Archaeological and historical records document material counting and administrative numeral systems in specific contexts.[2,4-10,15-18]

What We Still Don't Know

For most early marks, we do not know whether they represented quantity, which units they tracked, whether their rule was shared, or whether spoken number words accompanied them. We also cannot reconstruct the lost majority of perishable devices and gestures. Nor does present cognitive diversity give a direct chronology for when particular prehistoric communities coordinated exact words, objects and signs.

What Could Change Our Minds

Repeated marked objects from secure contexts, associated with independently known countable processes and governed by the same demonstrable grouping rule, would strengthen early numerical interpretations. New residue, use-wear or refitting evidence could weaken them. Decipherable intermediate records could clarify transitions among tokens, impressed signs and written numerals. Broader cognitive research could also revise how approximate, exact and embodied systems interact.

If You Remember Only One Thing

Quantity can be managed without numerals, but exact counting requires a learned mapping among units, order and cardinal value; a repeated ancient mark proves none of those mappings by itself.

Related Articles

KNO-001 · A Mark on a Stone KNO-003 · From Marks to Symbols KNO-004 · From Symbols to Writing KNO-007 · Calendars and Time Measurement KNO-011 · Measuring Before Theorising

References

- [1] Francesco d'Errico; Luc Doyon; Ivan Colagè; Alain Queffelec; Emma Le Vraux; Giacomo Giacobini; Bernard Vandermeersch; Bruno Maureille. (2018). From number sense to number symbols: an archaeological perspective. *Philosophical Transactions of the Royal Society B* 373, 20160518. DOI: 10.1098/rstb.2016.0518. <https://doi.org/10.1098/rstb.2016.0518> [2] Lisa Feigenson; Stanislas Dehaene; Elizabeth Spelke. (2004). Core systems of number. *Trends in Cognitive Sciences* 8, 307-314. DOI: 10.1016/j.tics.2004.05.002. <https://doi.org/10.1016/j.tics.2004.05.002> [3] Andreas Nieder. (2016). The neuronal code for number. *Nature Reviews Neuroscience* 17, 366-382. DOI: 10.1038/nrn.2016.40. <https://doi.org/10.1038/nrn.2016.40> [4] Peter Gordon. (2004). Numerical cognition without words: Evidence from Amazonia. *Science* 306, 496-499. DOI: 10.1126/science.1094492. <https://doi.org/10.1126/science.1094492> [5] Pierre Pica; Cathy Lemer; Véronique Izard; Stanislas Dehaene. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science* 306, 499-503. DOI: 10.1126/science.1102085. <https://doi.org/10.1126/science.1102085> [6] Michael C. Frank; Daniel L. Everett; Evelina Fedorenko; Edward Gibson. (2008). Number as a cognitive technology: Evidence from Pirahã language and cognition. *Cognition* 108, 819-824. DOI: 10.1016/j.cognition.2008.04.007. <https://doi.org/10.1016/j.cognition.2008.04.007> [7] Brian Butterworth; Robert Reeve; Fiona Reynolds; Delyth Lloyd. (2008). Numerical thought with and without words: Evidence from indigenous Australian children. *Proceedings of the National Academy of Sciences* 105, 13179-13184. DOI: 10.1073/pnas.0806045105. <https://doi.org/10.1073/pnas.0806045105> [8] Elizabet Spaepen; Marie Coppola; Elizabeth S. Spelke; Susan E. Carey; Susan Goldin-Meadow. (2011). Number without a language model. *Proceedings of the National Academy*

of Sciences 108, 3163-3168. DOI: 10.1073/pnas.1015975108. <https://doi.org/10.1073/pnas.1015975108>

[9] Karen Wynn. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology* 24, 220-251. DOI: 10.1016/0010-0285(92)90008-P. [https://doi.org/10.1016/0010-0285\(92\)90008-P](https://doi.org/10.1016/0010-0285(92)90008-P) [10] Barbara W. Sarnecka; Susan Carey. (2008). How counting represents number: What children must learn and when they learn it. *Cognition* 108, 662-674. DOI: 10.1016/j.cognition.2008.05.007. <https://doi.org/10.1016/j.cognition.2008.05.007> [11] Andrea Bender; Sieghard Beller. (2012). Nature and culture of finger counting: diversity and representational effects of an embodied cognitive tool. *Cognition* 124, 156-182. DOI: 10.1016/j.cognition.2012.05.005. <https://doi.org/10.1016/j.cognition.2012.05.005> [12] Geoffrey B. Saxe. (1981). Body parts as numerals: a developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development* 52, 306-316. DOI: 10.2307/1129244. <https://doi.org/10.2307/1129244> [13] Karenleigh A. Overmann. (2016). The role of materiality in numerical cognition. *Quaternary International* 405, 42-51. DOI: 10.1016/j.quaint.2015.05.026. <https://doi.org/10.1016/j.quaint.2015.05.026> [14] Karenleigh A. Overmann. (2018). Constructing a concept of number. *Journal of Numerical Cognition* 4, 464-493. DOI: 10.5964/jnc.v4i2.161. <https://doi.org/10.5964/jnc.v4i2.161> [15] Hans J. Nissen; Peter Damerow; Robert K. Englund. (1993). *Archaic Bookkeeping: Early Writing and Techniques of Economic Administration in the Ancient Near East*. University of Chicago Press. <https://cdli.earth/publications/1657148> [16] Denise Schmandt-Besserat. (1992). *Before Writing, Volume I: From Counting to Cuneiform*. University of Texas Press. <https://utpress.utexas.edu/9781477325766/> [17] Christopher Woods; Emily Teeter; Geoff Emberling (editors). (2010). *Visible Language: Inventions of Writing in the Ancient Middle East and Beyond*. Oriental Institute Museum Publications 32. <https://isac.uchicago.edu/research/publications/oimp/oimp-32-visible-language-inventions-writing-ancient-middle-east-and> [18] Stephen Chrisomalis. (2010). *Numerical Notation: A Comparative History*. Cambridge University Press. DOI: 10.1017/CBO9780511676062. <https://doi.org/10.1017/CBO9780511676062> [19] Jiajie Zhang; Donald A. Norman. (1994). Representations in distributed cognitive tasks. *Cognitive Science* 18, 87-122. DOI: 10.1207/s15516709cog1801_3. https://doi.org/10.1207/s15516709cog1801_3 [20] Karl Menninger. (1992). *Number Words and Number Symbols: A Cultural History of Numbers*. Dover Publications.