

The Hidden Boundary Problem: Understanding, Visualisation and Scientific Reasoning

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HEB-002 · Independent Technical Article · Version v0.2 · 22 September 2026

Abstract

This article treats definitions, scales, equations, diagrams and rules as compressed scientific representations. They are indispensable for reasoning and teaching, but they can become conceptual boundaries when their domain of validity is forgotten and the representation is treated as if it were the phenomenon itself. This article develops the hidden boundary problem: the failure that occurs when a useful simplification becomes an assumed law of nature or when a memorised procedure replaces understanding. The discussion uses retrospective educational examples, dimensional reasoning, redox balancing, and the visual interpretation of limits, derivatives and integrals to show how scientific knowledge can be reconstructed from connected principles rather than recalled as isolated rules. The article distinguishes observation from interpretation, intuition from evidence, analysis from synthesis, and representation from reality. It argues that conceptual visualisation is valuable not because pictures are intrinsically superior to equations, but because movement between verbal, graphical, dimensional, symbolic and physical representations exposes structure and constraints. The central methodological proposition is that connected understanding can make a result reconstructable from a smaller number of constraints and relations, rather than leaving it only as an isolated item to recall. Questioning is therefore treated not as rejection of established knowledge, but as disciplined testing of the boundaries within which a representation remains valid.

Keywords: scientific reasoning; conceptual understanding; visualisation; dimensional analysis; limits; derivative; integral; redox balancing; pH; scientific models; intuition; observation; interpretation; analysis; synthesis; education

1. Purpose and evidence boundary

This article is a methodological and educational reflection rather than a quantitative study of learning outcomes. It combines three kinds of material: first-person retrospective examples from the author's scientific education, elementary scientific demonstrations used only as conceptual illustrations, and documentary arguments about representation and observation developed in earlier 3Dresyns independent work. These classes should not be collapsed. A remembered examination episode is not archival evidence about a curriculum, and a useful pedagogical example is not evidence that all students learn in the same way.

The article advances one central proposition:

Understanding makes knowledge reconstructable; memorisation makes it retrievable.

The two are not opposites. Memory remains essential. The distinction is that understanding supplies constraints, relations and representations that can recover an idea even when its exact verbal or symbolic form has been forgotten.

2. The hidden boundary problem

Science needs boundaries. A measurement has a calibrated range. A model has assumptions. A scale has conventional reference points. An equation may be derived under limiting conditions. A classroom explanation removes complications so that an underlying relation can be seen. The problem begins when the boundary that organised learning is silently converted into a boundary of nature.

A useful rule can therefore fail in two directions. It can be used outside the conditions that justify it, or it can prevent the learner from recognising a valid case that lies outside the familiar classroom range. The danger is not simplification. The danger is forgetting that simplification occurred.

A boundary used to organise knowledge is not automatically a boundary of nature.

3. A remembered pH examination problem

The author recalls an entrance examination in chemistry, taken after secondary education, that included a very concentrated strong acid, approximately 10 mol L^{-1} . Using the familiar school-level relation $\text{pH} \approx -\log_{10}[\text{H}^+]$, the numerical result is approximately -1. The remembered difficulty was not the logarithm but the learned expectation that pH belonged between 0 and 14. Accepting the calculated negative value required treating the familiar interval as a teaching range rather than an inviolable wall.

This example is deliberately bounded. At very high concentrations, the simple concentration expression is not a complete thermodynamic treatment of pH. That qualification strengthens, rather than weakens, the methodological point: both the familiar 0-14 interval and the simplified concentration equation have domains. Scientific maturity consists partly in recognising those domains.

rule + domain of validity $\neq$ universal law

4. Observation is not interpretation

Observation supplies constrained contact with a phenomenon, but observation does not arrive already interpreted. A curve on a screen, a colour change, a measured voltage, a spectrum or a numerical output becomes scientifically meaningful only through a framework that identifies what was measured and why it matters. HEB-001 made an analogous distinction for laboratory measurement: a numerical output is not automatically

a valid measurement, because instrument, specimen, reference, environment, operator, protocol and analysis act together. The same discipline applies conceptually. A representation is not automatically the reality it represents. [1]

The Maxwell study provides a historical illustration. Its documentary conclusion is not that imagination can replace evidence, but that representations become scientifically productive when they encounter equations, measurements and possible failure. Maxwell constructed representations, calculated consequences, compared them with observation or measurement, and revised mechanisms when constraints changed. [2]

5. Perception, intuition, analysis, synthesis and questioning

For the purposes of this article, scientific reasoning is represented as an iterative interaction among several operations. Observation supplies evidence. Perception selects patterns and contrasts. Intuition proposes possible structure. Interpretation assigns meaning. Analysis separates a problem into components. Synthesis reconnects those components into a coherent whole. Questioning tests whether the chosen representation has become too narrow. Evidence then constrains which interpretations survive.

intuition → question → hypothesis → test → evidence

The arrow should not be shortened to intuition → truth. Intuition is valuable because it generates questions and candidate structures, not because it is self-validating. Likewise, questioning is not equivalent to rejecting established knowledge. A scientifically useful question seeks the assumptions, limits and consequences of a claim.

6. Dimensional reasoning: when structure survives memory failure

A simple example comes from dimensional analysis. A student may forget the exact form of an equation while retaining an understanding that a physically meaningful equality must be dimensionally consistent. That structural constraint can eliminate impossible expressions and sometimes reconstruct much of a relation.

Energy is particularly useful because different physical descriptions converge on the same dimensions:

$$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

$$E_k = \frac{1}{2}mv^2 \rightarrow \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

$$E_p = mgh \rightarrow \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

$$W = Fd \rightarrow \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

Dimensional reasoning cannot determine everything. It does not generally recover dimensionless constants such as the 1/2 in kinetic energy, the sign of a term, or an arbitrary functional form. Its value is different: it turns units into information about physical structure.

Memory retrieves an equation. Understanding constrains what the equation can be.

7. Units as physical meaning: gravitational acceleration

The notation $\text{m} \cdot \text{s}^{-2}$ can be opaque when learned only as a unit to be copied. Near the Earth's surface, the familiar value $g \approx 9.8 \text{ m} \cdot \text{s}^{-2}$ can instead be read as approximately 9.8 metres per second of velocity change, per second. The squared time dimension is no longer typographical decoration:

$$\text{m} \cdot \text{s}^{-2} = (\text{m} \cdot \text{s}^{-1}) \text{ per second}$$

A compact symbolic expression has now been converted into a verbal and physical representation. This movement between representations is used here as one operational sign of connected understanding; it is not proposed as a validated psychometric measure.

8. Conceptual visualisation: from finite change to the derivative

Calculus provides a particularly clear example of the difference between symbolic familiarity and conceptual understanding. Consider a curve $y = f(x)$. Between two separated points the finite changes are Δx and Δy , and their ratio $\Delta y/\Delta x$ is the average rate of change over that interval. Geometrically, it is the slope of the secant joining the points.

The derivative asks a more difficult question: what is the rate of change at one point? We do not set Δx equal to zero and divide by zero. Instead, we examine the value approached by the difference quotient as the interval shrinks:

$$f'(x) = \lim(\Delta x \rightarrow 0) [f(x+\Delta x)-f(x)]/\Delta x = dy/dx$$

Visually, the second point moves toward the first and the secant tends toward the tangent. The limit provides a rigorous way to reason about behaviour at a boundary without pretending that a finite interval has literally become zero.

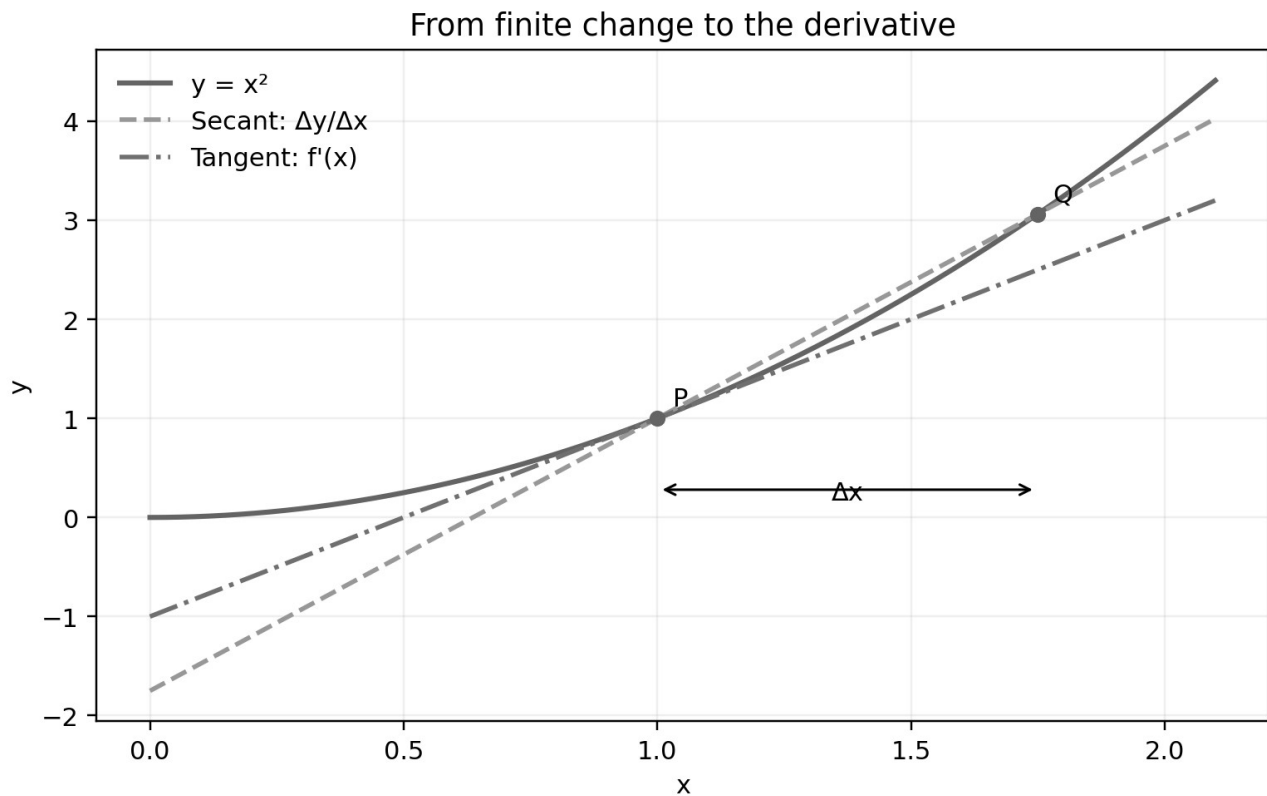


Figure 1. Visual interpretation of the derivative. A finite secant slope $\Delta y/\Delta x$ becomes a local tangent slope in the limit as Δx tends to zero.

9. From finite accumulation to the integral

The complementary idea begins with accumulation. Over a finite interval, the area under a non-negative curve can be approximated by rectangles of width Δx and height $f(x_i)$. Each rectangle contributes approximately $f(x_i)\Delta x$, and the sum of all contributions is a finite approximation:

$$A \approx \sum f(x_i)\Delta x$$

Refining the partition makes the rectangles narrower. In the limit, under the usual conditions for integrability, the finite sums converge to the definite integral:

$$\int_a^b f(x) \, dx = \lim(\max \Delta x_i \rightarrow 0) \sum f(x_i)\Delta x_i$$

For non-negative functions this has the familiar geometric interpretation as area under the curve. More generally, the definite integral represents signed accumulation. The important conceptual pair is therefore not merely “slope and area”, but local change and accumulated contribution.

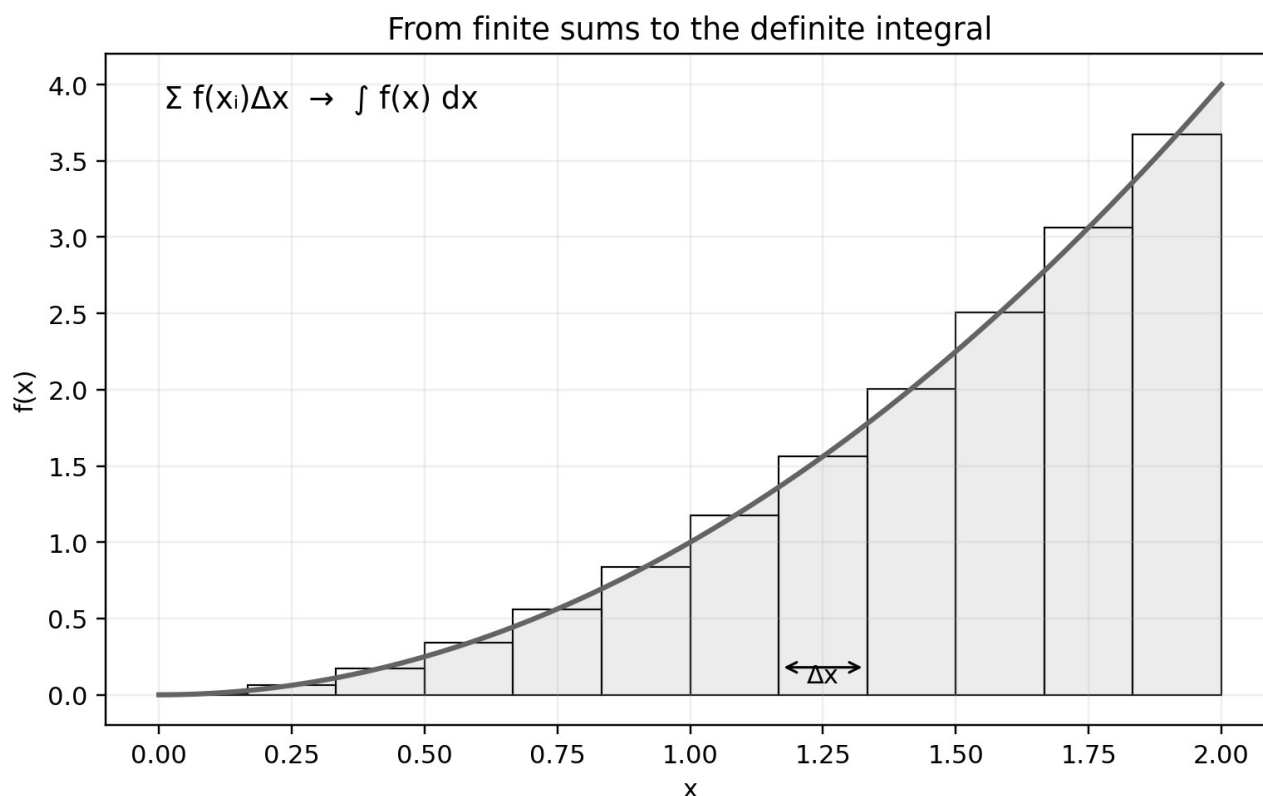


Figure 2. Visual interpretation of the definite integral. Finite rectangular contributions are refined until their limiting sum represents the accumulated quantity.

10. Derivative and integral as analysis and synthesis

The mathematical relation between differentiation and integration is formalised by the fundamental theorem of calculus under appropriate conditions. At the conceptual level used here, they also provide a useful epistemological analogy. Differentiation asks what is happening locally; integration asks what emerges when local contributions are accumulated.

differentiation: whole curve $\rightarrow$ local change

integration: local contributions $\rightarrow$ accumulated whole

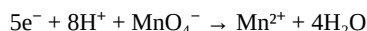
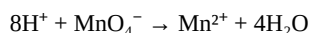
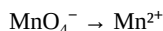
This is an analogy, not an identity between mathematics and cognition. Its value is that a learner can see analysis and synthesis in a mathematical object rather than memorise two disconnected sets of symbols.

11. Redox balancing as conservation rather than recipe

Another example arises in early chemistry education. Redox balancing can be taught as a sequence of apparently arbitrary instructions: add water, add protons, add hydroxide, add electrons, then simplify. The procedure becomes more intelligible when every operation is tied to conservation.

For an aqueous half-reaction, the student can treat the problem as two coupled accounts: matter and charge. Water and H^+ are bookkeeping species for oxygen and hydrogen in acidic medium; electrons close the charge balance. In basic medium, OH^- can be introduced symmetrically to neutralise H^+ and the resulting water simplified.

For example, the reduction of permanganate in acidic medium can be reconstructed step by step:



At the end, both atoms and total charge are conserved. The method is easier to remember because each operation answers a visible imbalance. Comprehension converts an arbitrary sequence into a constrained reconstruction.

12. Multiple representations as a test of understanding

A concept becomes more robust when it can be translated between representations without losing its structure. The following transformations recur throughout the examples above:

Representation	Question it answers	What understanding adds
Verbal	Can the idea be explained without symbols?	Reveals causal or physical meaning.
Symbolic	Can the relation be written compactly?	Provides precision and calculational power.
Graphical	Can the relation be seen as shape or change?	Exposes trends, limits and geometry.
Dimensional	Are the physical quantities compatible?	Constrains possible equations.
Procedural	Can the result be reconstructed step by step?	Connects rules to necessity.
Experimental	What observation could support or challenge it?	Reconnects representation to reality.

A formula remembered is information. A formula that can be visualised, interpreted, tested and reconstructed has become understanding.

13. Understanding as cognitive compression

The term compression is used here as an epistemological metaphor rather than a measured cognitive quantity. A learner who stores every scientific relation as an isolated item requires a large catalogue of independent memories. A learner who recognises a smaller set of connected principles can derive or constrain many of those items when needed. Conservation, dimensional consistency, symmetry, energy relationships, limits and charge balance are examples of such organising principles.

The most durable mnemonic is often not an arbitrary phrase but a structure that makes the answer necessary. This does not eliminate memory. It gives memory anchors and routes of reconstruction.

14. The boundary-perturbation principle

HEB-001 proposed deliberately perturbing a measurement system before trusting it. A related principle can be applied to conceptual models. When a rule appears absolute, perturb the boundary mentally and ask which assumption breaks first. What happens outside the usual range? Which variable was held fixed? Is the category intrinsic to nature or introduced for convenience? Can the representation be translated into another form?

This article proposes the following boundary-perturbation principle:

Before treating a learned boundary as natural, test what happens when the system approaches, crosses or redefines that boundary.

The pH example asks whether a familiar numerical interval is a wall. The derivative asks what happens as a finite interval approaches zero. Dimensional reasoning asks whether an equation remains physically coherent when its symbols are rearranged. Redox balancing asks what quantities must remain conserved regardless of the chosen bookkeeping route.

15. Implications for scientific education

Scientific teaching cannot avoid simplification. The proposal made here is narrower than a claim about measured learning outcomes: teach the status of a representation alongside the representation itself. A rule can be presented with the question it answers, the assumptions that make it useful, and at least one route by which it can be reconstructed or challenged.

- Teach units as physical relations, not suffixes attached to numbers.
- Pair symbolic calculus with geometric change and accumulation.
- Present redox balancing as conservation of atoms and charge before presenting the algorithm.
- Distinguish classroom ranges from physical impossibility.
- Ask learners to translate the same concept between words, equations, graphs and physical interpretation.
- Treat intuition as a source of questions and evidence as the constraint on answers.

16. Implications for research and engineering

The same habits matter after formal education. Engineering failures often arise because a convenient model, specification, instrument range or historical procedure is used outside its original domain. Conceptual flexibility is therefore not merely educational. It is part of technical risk control. A scientist or engineer who can reconstruct why a rule works is better positioned to recognise when the rule has stopped working.

The strongest practice combines representation with perturbation: construct a model, calculate its consequences, compare them with observation, disturb the assumptions, and revise the model when the constraints change. This sequence parallels the documentary pattern developed in the Maxwell study and the measurement-system discipline developed in HEB-001. [1,2]

17. Conclusions

1. Scientific representations are necessary, but their domains should not be mistaken for boundaries of nature.
2. Memorisation and understanding are complementary; understanding adds constraints that make knowledge reconstructable.
3. Dimensional analysis converts units into structural information and can reject impossible equations even when exact formulas are forgotten.
4. Visualisation of limits makes the derivative intelligible as local change and the integral intelligible as accumulated contribution.
5. Redox balancing becomes easier to reconstruct when treated as conservation of matter and charge rather than as an arbitrary recipe.
6. Observation, interpretation, intuition, analysis, synthesis and questioning play different roles and should not be collapsed into one process.
7. Questioning is scientifically productive when it tests assumptions and boundaries rather than merely rejecting established results.
8. Deep understanding is demonstrated by the ability to move among verbal, symbolic, graphical, dimensional, procedural and experimental representations.
9. Within this methodological framework, an educational objective can include not only correct recall but also the ability to reconstruct, test and revise a scientific representation.

The broader conclusion is:

Connected understanding can turn isolated rules into consequences of stated principles and constraints.

References

[1] Segurola, J. HEB-001 - The Hidden Error Budget: Measurement-System Failure, False Precision and Experimental Design in Laboratory Science. 3Dresyns Independent Technical Articles, v1.0, 20 September 2026.

[2] Segurola, J. Maxwell: Thinking the Invisible - James Clerk Maxwell and the Art of Reasoning Beyond What Could Be Seen. Version 1.3, 11 September 2026. DOI: 10.5281/zenodo.22714429.

Knowledge boundary

The educational episodes described as first-person recollections are retrospective and are not presented as documentary evidence about a particular examination board, teacher or curriculum. The calculus, dimensional-analysis and redox examples are elementary conceptual demonstrations, not new mathematical or chemical results. The Maxwell discussion is limited to the documentary interpretation already developed in the cited book. No claim is made here that one pedagogical representation is universally optimal for all learners.

Limitations

No controlled educational experiment was performed. Learning gains, retention, transfer and age-specific cognitive effects were not measured. The term cognitive compression is used metaphorically and is not presented as a quantified psychological construct. The article therefore proposes a framework for scientific reasoning and teaching, not a validated learning theory.